## Recent Research on Search Based Software Testing: Part 2



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Test Suite Minimization

Problem Formulation Landscape Theory Decomposition SAT Transf. Results

Software Product Pairwise Prioritized
Lines
Testing in SPL

# Test Suite Mimimization in Regression Testimg 

## F. Arito et al., SSBSE 2012

Test Suite Minimization

## Test Suite Minimization

## Given:

$>$ A set of test cases $T=\left\{t_{1}, t_{2}, \ldots, t_{n}\right\}$
$>$ A set of program elements to be covered (e.g., branches) $E=\left\{e_{1}, e_{2}, \ldots, e_{k}\right\}$
$>$ A coverage matrix

$\mathbf{M}=$|  | $\mathbf{e}_{1}$ | $\mathbf{e}_{2}$ | $\mathbf{e}_{3}$ | $\ldots$ | $\mathbf{e}_{k}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $t_{1}$ | 1 | 0 | 1 | $\ldots$ | 1 |
| $t_{2}$ | 0 | 0 | 1 | $\ldots$ | 0 |
| $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ |
| $t_{n}$ | 1 | 1 | 0 | $\ldots$ | 0 | $m_{i j}= \begin{cases}1 & \text { if element } e_{j} \\ 0 & \text { otherwise covered by test } t_{i}\end{cases}$

Find a subset of tests $X \subseteq T$ maximizing coverage and minimizing the testing cost

$$
\text { minimize } \quad \operatorname{cost}(X)=\sum_{\substack{i=1 \\ t_{i} \in X}}^{n} c_{i}
$$

## Yoo \& Harman

 maximize $\operatorname{cov}(X)=\mid\left\{e_{j} \in \mathcal{E} \mid \exists t_{i} \in X\right.$ with $\left.m_{i j}=1\right\} \mid$
## NP-hard Problems

In many papers we can read...
"Our optimization problem is NP-hard, and for this reason we use...
$\left\{\begin{array}{ll}\cdot & \text { Metaheuristic techniques } \\ \cdot & \text { Heuristic algorithms } \\ \cdot & \text { Stochastic algorithms }\end{array}\right\}$
... which do not ensure an optimal solution but they are able to find good solutions in a reasonable time."

As far as we know: no efficient (polynomial time) algorithm exists for solving NP-hard problems

But we know "inefficient" algorithms (exponential time in the worst case)

## The SATisfiability Problem

Can we find an assignment of boolean values (true and false) to the variables such that all the formulas are satisfied?

$$
\begin{aligned}
& \neg A \wedge(B \vee C) \\
& (A \vee B) \wedge(\neg B \vee C \vee \neg D) \wedge(D \vee \neg E) \\
& A \vee B
\end{aligned}
$$

The first NP-complete problem (Stephen Cook, 1971)
If it can be solved efficiently (polynomial time) then $\mathrm{P}=\mathrm{NP}$
The known algorithms solve this problem in exponential time (worst case)

## State-of-the-art algorithms in SAT

Nowadays, SAT solvers can solve instances with 500000 boolean variables
This means a search space of $2^{500} 000 \approx 10^{150514}$

## The SATisfiability Problem

Main research question:

## Can we use the advances of SAT solvers to solve optimization algorithms up to optimality?

My favourite problem

Test Suite Minimization

SAT instance


Use SAT solvers

Optimal solution

## Experimental Results

## Outline

## Original TSM Instance

## SAT <br> Instance

## Pseudo-Boolean Constraints

A Pseudo-Boolean (PB) constraint has the form:

$$
\sum_{i=1}^{n} a_{i} x_{i} \odot B
$$

where

$$
\begin{aligned}
& \odot \in\{<, \leq,=, \neq,>, \geq\} \\
& a_{i}, B \in \mathbb{Z} \quad x_{i} \in\{0,1\}
\end{aligned}
$$

Can be translated to SAT instances (usually efficient)
Are a higher level formalism to specify a decision problem
Can be the input for MiniSAT+

## Translating Optimization to Decision Problems

Let us assume we want to minimize $f(x)$


Optimal solution found
The same can be done with multi-objective problems, but we need more PB constraints

$$
f_{1}(y) \leq B_{1} \quad f_{2}(y) \leq B_{2} \quad \cdots \quad f_{m}(y) \leq B_{m}
$$

## PB Constraints for the TSM Problem



Cost
$\sum_{i=1}^{n} c_{i} t_{i} \leq B$

Coverage

$$
\sum_{j=1}^{m} e_{j} \geq P
$$

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## Example

|  | $e_{1}$ | $e_{2}$ | $e_{3}$ | $e_{4}$ |
| :---: | :---: | :---: | :---: | :---: |
| $t_{1}$ | 1 | 0 | 1 | 0 |
| $t_{2}$ | 1 | 1 | 0 | 0 |
| $t_{3}$ | 0 | 0 | 1 | 0 |
| $t_{4}$ | 1 | 0 | 0 | 0 |
| $t_{5}$ | 1 | 0 | 0 | 1 |
| $t_{6}$ | 0 | 1 | 1 | 0 |

Bi-objective problem

$$
\left\{\begin{array}{ccc}
e_{1} \leq & t_{1}+t_{2}+t_{4}+t_{5} & \leq 6 e_{1} \\
e_{2} \leq & t_{2}+t_{6} & \leq 6 e_{2} \\
e_{3} \leq & t_{1}+t_{3}+t_{6} & \leq 6 e_{3} \\
e_{4} \leq & t_{5} & \leq 6 e_{4} \\
t_{1}+t_{2}+t_{3}+t_{4}+t_{5}+t_{6} & \leq B \\
e_{1}+e_{2}+e_{3}+e_{4} & \geq P
\end{array}\right.
$$

Single-objective problem (total coverage)

$$
\begin{aligned}
t_{1}+t_{2}+t_{4}+t_{5} & \geq 1 \\
t_{2}+t_{6} & \geq 1 \\
t_{1}+t_{3}+t_{6} & \geq 1 \\
t_{5} & \geq 1
\end{aligned}
$$

$$
t_{1}+t_{2}+t_{3}+t_{4}+t_{5}+t_{6} \leq B
$$

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## Algorithm for Solving the 2-obj TSM

## Total coverage



With coverage=|E| increase cost until success

Decrease cost and find the maximum coverage again
and again

Cost

## TSM Instances

Instances from the Software-artifact Infrastructure Repository (SIR) http://sir.unl.edu/portal/index.php

| Instance | Tests | Elements to cover |
| :--- | ---: | ---: |
| printtokens | 4130 | 195 |
| printtokens2 | 4115 | 192 |
| replace | 5542 | 208 |
| schedule | 2650 | 126 |
| schedule2 | 2710 | 119 |
| tcas | 1608 | 54 |
| totinfo | 1052 | 117 |

Cost of each test: 1

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## Pareto Front

Pareto front


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## Pareto Front

| Instance | Elements Tests Coverage |  |  | Solution |
| :---: | :---: | :---: | :---: | :---: |
| printtokens | 195 | 5 | 100\% | $\left(t_{2222}, t_{2375}, t_{3438}, t_{4100}, t_{4101}\right)$ |
|  | 194 | 4 | 99.48\% | $\left(t_{1908}, t_{2375}, t_{4099}, t_{4101}\right)$ |
|  | 192 | 3 | 98.46\% | $\left(t_{1658}, t_{2363}, t_{4072}\right)$ |
|  | 190 | 2 | 97.43\% | $\left(t_{1658}, t_{3669}\right)$ |
|  | 186 | 1 | 95.38\% | $\left(t_{2597}\right)$ |
| printtokens2 | 192 | 4 | 100\% | $\left(t_{2521}, t_{2526}, t_{4085}, t_{4088}\right)$ |
|  | 190 | 3 | 98.95\% | $\left(t_{457}, t_{3717}, t_{4098}\right)$ |
|  | 188 | 2 | 97.91\% | $\left(t_{2190}, t_{3282}\right)$ |
|  | 184 | 1 | 95.83\% | $\left(t_{3717}\right)$ |
| replace | 208 | 8 | 100\% | $\left.t_{410}, t_{653}, t_{1279}, t_{1301}, t_{3134}, t_{4057}, t_{4328}\right)$ |
|  | 207 | 7 | 99.51\% | $\left(t_{309}, t_{358}, t_{653}, t_{776}, t_{1279}, t_{1795}, t_{3248}\right)$ |
|  | 206 | 6 | 99.03\% | $\left(t_{275}, t_{290}, t_{1279}, t_{1938}, t_{2723}, t_{2785}\right)$ |
|  | 205 | 5 | 98.55\% | $\left(t_{426}, t_{1279}, t_{1898}, t_{2875}, t_{3324}\right)$ |
|  | 203 | 4 | 97.59\% | $\left(t_{298}, t_{653}, t_{3324}, t_{5054}\right)$ |
|  | 200 | 3 | 96.15\% | $\left(t_{2723}, t_{2901}, t_{3324}\right)$ |
|  | 195 | 2 | 93.75\% | $\left(t_{358}, t_{5387}\right)$ |
|  | 187 | 1 | 89.90\% | $\left(t_{358}\right)$ |
| schedule | 126 | 3 | 100\% | $\left(t_{1403}, t_{1559}, t_{1564}\right)$ |
|  | 124 | 2 | 98.41\% | $\left(t_{1570}, t_{1595}\right)$ |
|  | 122 | 1 | 96.82\% | $\left(t_{1572}\right)$ |
| schedule2 | 119 | 4 | 100\% | $\left(t_{2226}, t_{2458}, t_{2462}, t_{2681}\right)$ |
|  | 118 | 3 | 99.15\% | $\left(t_{101}, t_{1406}, t_{2516}\right)$ |
|  | 117 | 2 | 98.31\% | $\left(t_{2461}, t_{2710}\right)$ |
|  | 116 | 1 | 97.47\% | $\left(t_{1584}\right)$ |
| tcas | 54 | 4 | 100\% | $\left(t_{5}, t_{1191}, t_{1229}, t_{1608}\right)$ |
|  | 53 | 3 | 98.14\% | $\left(t_{13}, t_{25}, t_{1581}\right)$ |
|  | 50 | 2 | 92.59\% | $\left(t_{72}, t_{1584}\right)$ |
|  | 44 | 1 | 81.48\% | $\left(t_{217}\right)$ |
| totinfo | 117 | 5 | 100\% | $\left(t_{62}, t_{118}, t_{218}, t_{1000}, t_{1038}\right)$ |
|  | 115 | 4 | 98.29\% | $\left(t_{62}, t_{118}, t_{913}, t_{1016}\right)$ |
|  | 113 | 3 | 96.58\% | $\left(t_{65}, t_{216}, t_{913}\right)$ |
|  | 111 | 2 | 94.87\% | $\left(t_{65}, t_{919}\right)$ |
|  | 110 | 1 | 94.01\% | $\left(t_{179}\right)$ |

## Reduction in the Number of Test Cases

Since we are considering cost 1 for the tests, we can apply an a priori reduction in the original test suite

|  | $\mathbf{e}_{1}$ | $\mathbf{e}_{2}$ | $\mathbf{e}_{3}$ | $\ldots$ | $\mathbf{e}_{m}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $t_{1}$ | 1 | 0 | 0 | $\ldots$ | 1 |
| $t_{2}$ | 1 | 0 | 1 | $\ldots$ | 1 |
| $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ |
| $t_{n}$ | 1 | 1 | 0 | $\ldots$ | 0 |


| Instance | Original Size | Reduced Size | Elements to cover |
| :--- | ---: | ---: | ---: |
| printtokens | 4130 | 40 | 195 |
| printtokens2 | 4115 | 28 | 192 |
| replace | 5542 | 215 | 208 |
| schedule | 2650 | 4 | 126 |
| schedule2 | 2710 | 13 | 119 |
| tcas | 1608 | 5 | 54 |
| totinfo | 1052 | 21 | 117 |

## Results with the Reduction

The optimal Pareto Front for the reduced test suite can be found from 200 to 180000 times faster

|  | Original (s) Reduced (s) |  |
| :--- | ---: | ---: |
| printtokens | 3400.74 | 2.17 |
| printtokens2 | 3370.44 | 1.43 |
| replace | 1469272.00 | 345.62 |
| schedule | 492.38 | 0.24 |
| schedule2 | 195.55 | 0.27 |
| tcas | 73.44 | 0.33 |
| totinfo | 181823.50 | 0.96 |

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## Solitware Product Lines Testing

R. Lopez-Herrejon et al., ICSM 2013

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## Software Product Lines

A product line is a set of related products developed from a shared set of assets

- The products have similar characteristics
- The products have unique characteristics

Advantages

- Support customization
- Improves reuse
- Reduce time to market


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## Background Testing SAT Transform. Results

## Software Product Lines

In Software Product Lines the product is Software

## They are modelled using Feature Models



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## Feature Models



Cross-tree constraints

> Graph Product Line Feature Model

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## Background Testing SAT Transform. Results

## Testing of Software Product Lines

The GPL Feature Model is small: 73 distinct products


But the number of products grows exponentially with the number of features...

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... and testing each particular product is not viable

## Testing of SPLs: Combinatorial Interaction Testing

Assuming each feature has been tested in isolation, most of the defects come from the interaction between features

Combinatorial Interaction Testing consists in selecting the minimum number of products that covers all $t$-wise interactions ( $t$-wise coverage).


## Testing of SPLs: Multi-Objective Formulation

If we don't have the resources to run all the tests, which one to choose?

Multi-objective formulation:
minimize the number of products maximize the coverage (t-wise interactions)

The solution is not anymore a table of products, but a Pareto set


## GPL

2-wise interactions

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## Testing of SPLs: Approach

Original


Instance

## Background Testing SAT Transform. Results

## Testing of SPLs: Approach

Modelling SPLT using PseudoBoolean constraints

| Variable | Meaning |
| :--- | :--- |
| $x_{p, i}$ | Presence of feature $i$ in product $p$ |
| $c_{p, i, j, k}$ | Product $p$ covers the pair $(i, j)$ with signature $k$ |
| $d_{i, j, k}$ | The pair $(i, j)$ with signature $k$ is covered by some product |
| takes values $0,1,2$ and 3. |  |

All the variables are boolean $\{0,1\}$
The values of the signature are:

- 00 (both unselected)
- 10 (only first selected)
- 01 (only second selected)
- 11 (both selected)


## Background Testing SAT Transform. Results

## Testing of SPLs: Approach

Equations of the model

- For each product $p$
- Constraints imposed by the Feature Model
- For each product $p$ and pair of features $i$ and $j$

$$
\begin{aligned}
& 2 c_{p, i, j, 3} \leq x_{p, i}+x_{p, j} \leq 1+c_{p, i, j, 3} \\
& 2 c_{p, i, j, 2} \leq x_{p, i}+\left(1-x_{p, j}\right) \leq 1+c_{p, i, j, 2} \\
& 2 c_{p, i, j, 1} \leq\left(1-x_{p, i}\right)+x_{p, j} \leq 1+c_{p, i, j, 1} \\
& 2 c_{p, i, j, 0} \leq\left(1-x_{p, i}\right)+\left(1-x_{p, j}\right) \leq 1+c_{p, i, j, 0}
\end{aligned}
$$

## Background Testing SAT Transform. Results

## Testing of SPLs: Approach

Equations of the model (cont.)

- For each pair of features $i$ and $j$ and signature $k$

$$
d_{i, j, k} \leq \sum_{p} c_{p, i, j, k} \leq n d_{i, j, k}
$$

- $n$ is the number of products
- Objective: maximize coverage

$$
\max : \sum_{i, j, k} d_{i, j, k}
$$

## Testing of SPLs: Approach

Algorithm 1 Algorithm for obtaining the optimal Pareto set.
optimal_set $\leftarrow\{\emptyset\}$;
$\operatorname{cov}[0] \leftarrow 0$;
$\operatorname{cov}[1] \leftarrow C_{2}^{f}$;
sol $\leftarrow \operatorname{arbitrary}$ ValidSolution $(\mathrm{fm})$;
$i \leftarrow 1$;
while $\operatorname{cov}[i] \neq \operatorname{cov}[i-1]$ do
optimal_set $\leftarrow$ optimal_set $\cup\{$ sol $\}$;
$i \leftarrow i+1 ;$
$m \leftarrow$ prepareMathModel $(f m, i)$;
sol $\leftarrow$ solveMathModel $(m)$;
$\operatorname{cov}[i] \leftarrow|\operatorname{sol}| ;$
end while

## Testing of SPLs: Results

Experiments on 118 feature models taken from
SPLOT repository (http://www.splot-research.org)
SPL Conqueror (http://wwwiti.cs.uni-magdeburg.de/~nsiegmun/SPLConqueror/)


## 16 to 640 products

## Intel Core2 Quad Q9400

2.66 GHz, 4 GB

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## Prioritized Pairwise

 Testing im Software Product LimesR. Lopez-Herrejon et al., GECCO 2014

## Our contributions

- Formalization of prioritization testing scheme proposed by Johansen et al.
- Implementation with the Parallel Prioritized product line Genetic Solver (PPGS)
- Comprehensive evaluation and comparison against greedy approach.


## Prioritization Motivation

- Key ideas
- Each feature combination represents an important product of the SPL
- For each relevant product give a positive integer value that reflects the priority of the product
- Market importance
- Implementation costs
- ...


## Feature List and Feature Set

Definition 1. Feature List (FL) is the list of features in a feature model.
Definition 2. Feature Set (FS) is a 2-tuple [sel, $\overline{s e l}]$ where sel and $\overline{\text { sel }}$ are respectively the set of selected and not-selected features of a member product. Let FL be a feature list, thus sel, $\overline{\mathrm{sel}} \subseteq F L$, sel $\cap \overline{\mathrm{sel}}=\emptyset$, and sel $\cup \overline{\mathrm{sel}}=F L$. The terms p.sel and p.sel respectively refer to the set of selected and unselected features of product $p$.

- Example Feature List (FL)

Aircraft, Wing, Engine, Materials, High, Shoulder, Low, Piston, Jet, Metal, Wood, Plastic, Cloth

## Feature Set Example



Selected $=$ \{Aircraft, Wing, High, Engine, Piston, Materials, Cloth $\}$
Unselected $=$ \{Shoulder, Low, Jet, Metal, Wood, Plastic $\}$

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## Terminology (3)

Definition 3. A feature set $f s$ is valid in feature model fm, i.e. valid(fs, fm) holds, iff fs does not contradict any of the constraints introduced by fm.

- Examples of valid feature sets
- Aircraft, Wing, Engine, Materials, High, Shoulder, Low, Piston, Jet, Metal, Wood, Plastic, Cloth

| Prod | A | Wi | E | Ma | H | S | L | Pi | J | Me | Wo | Pl | C |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| p0 | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ |  |  | $\checkmark$ |  |  |  | $\checkmark$ |  |
| p1 | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ |  |  | $\checkmark$ |  |  |  |  | $\checkmark$ |
| p2 | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ |  |  |  | $\checkmark$ |  |  | $\checkmark$ |  |
| p3 | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ |  |  |  | $\checkmark$ |  |  |  | $\checkmark$ |
| p4 | $\checkmark$ | $\checkmark$ |  | $\checkmark$ | $\checkmark$ |  |  |  |  |  | $\checkmark$ |  |  |
| p5 | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ |  |  | $\checkmark$ |  |  | $\checkmark$ | 315 valid <br> 3eature sets |  |
| p6 | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ |  |  | $\checkmark$ |  | $\checkmark$ |  | featu |  |
| p7 | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ |  |  |  | $\checkmark$ | $\checkmark$ |  |  |  |

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## Background Algorithm Results

## Prioritized Product

Definition 4. A prioritized product pp is a 2-tuple [fs, w], where $f s$ represents a valid feature set in feature model $f m$ and $w \in \mathbb{R}$ represents its weight. Let $p p_{i}$ and $p p_{j}$ be two prioritized products. We say that $p p_{i}$ has higher priority than $p p_{j}$ for test-suite generation iff $p p_{i}$ 's weight is greater than $p p_{j}$ 's weight, that is $p p_{i} \cdot w>p p_{j} . w$.

## - Example

| Prod | A | Wi | E | Ma | H | S | L | Pi | J | Me | Wo | Pl | C |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| P $^{0}$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ |  |  | $\boxed{l}$ |  |  |  |  |  |
| pl | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ |  |  | $\checkmark$ |  |  |  |  | $\checkmark$ |

$$
\text { pp1 }=[p 1,17]
$$

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## Background Algorithm Results

## Pairwise configuration

Definition 5. A pairwise configuration pc is a 2-tuple [sel, $\overline{s e l}]$ representing a partially configured product, defining the selection of 2 features of feature list $F L$, i.e. pc.sel $\cup$ pc.sel $\subseteq$ $F L \wedge$ pc.sel $\cap p c . \overline{s e l}=\emptyset \wedge|p c . s e l \cup p c . \overline{s e l}|=2$. We say a pairwise configuration $p c$ is covered by feature set $f s$ iff pc.sel $\subseteq f$ s.sel $\wedge$ pc. $\overline{s e l} \subseteq f$ s. $\overline{s e l}$.

| Prod | A | Wi | E | Ma | H | S | L | Pi | J |  | 240 pairwise |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| p0 | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ |  |  | $\checkmark$ |  |  |  |  |  |
| DI | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ |  |  | $\checkmark$ |  |  |  |  |  |
| p2 | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ |  |  |  | $\checkmark$ |  |  |  |  |
| 05 | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ |  |  |  | $\checkmark$ |  |  |  |  |
| p4 | $\checkmark$ | $\checkmark$ |  | $\checkmark$ | $\checkmark$ |  |  |  |  |  | $\checkmark$ |  |  |
| p5 | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ |  |  | $\checkmark$ |  |  | $\checkmark$ |  |  |
| p6 | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ |  |  | $\checkmark$ |  | $\checkmark$ |  |  |  |
| p7 | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ |  |  |  | $\checkmark$ | $\checkmark$ |  |  |  |

pc1 $=[\{$ Plastic $\},\{$ Cloth $\}]$
pc2=[\{High, Wood\},\{\}]

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Background Algorithm Results

## Weighted Pairwise Configuration

Definition 6. A weighted pairwise configuration wpc is a 2tuple [pc,w] where pc is a pairwise configuration and $w \in \mathbb{R}$ represents its weight computed as follows. Let PP be a set of prioritized products and $P P_{p c}$ be a subset, $P P_{p c} \subseteq P P$, such that $P P_{p c}$ contains all prioritized products in $P P$ that cover $p c$ of $w p c$, i.e. $P P_{p c}=\{p p \in P P \mid p p . f s$ covers wpc. $p c\}$. Then $w=\sum_{p \in P P_{p c}} p . w$
pc1=[\{Plastic $\},\{$ Cloth $\}]$

| Prod | A | Wi | E | Ma | H | S | L | Pi | J | Me | Wo | Pl | C | weights |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| p0 | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ |  |  | $\checkmark$ |  |  |  | $\checkmark$ |  | (17) |
| 1 | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ |  |  | $\checkmark$ |  |  |  |  | $\checkmark$ | 17 |
| p2 | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ |  |  |  | $\checkmark$ |  |  | $\checkmark$ |  | (15) |
| ps | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ |  |  |  | $\checkmark$ |  |  |  | $\checkmark$ | 15 |
| p4 | $\checkmark$ | $\checkmark$ |  | $\checkmark$ | $\checkmark$ |  |  |  |  |  | $\checkmark$ |  |  | 13 |
| p5 | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ |  |  | $\checkmark$ |  |  | $\checkmark$ |  |  | 13 |
| p6 | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ |  |  | $\checkmark$ |  | $\checkmark$ |  |  |  | 6 |
| p7 | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ |  |  |  | $\checkmark$ | $\checkmark$ |  |  |  | 6 |

Test Suite Minimization

## Prioritized Pairwise Covering Array

Definition 7. A prioritized pairwise covering array ppCA for a feature model fm and a set of weighted pairwise configurations WPC is a set of valid feature sets FS that covers all weighted pairwise configurations in WPC whose weight is greater than zero: $\forall w p c \in W P C$ (wpc. $w>0 \Rightarrow \exists f s \in$ $p p C A$ such that fs covers wpc.pc).

- Example of ppCA
\(\left.\begin{array}{|c|c|c|c|c|c|c|c|c|c|c|c|c|}\hline A \& Wi \& E \& Ma \& H \& S \& L \& Pi \& J \& Me \& Wo \& Pl \& C <br>
\hline \hline \checkmark \& \checkmark \& \checkmark \& \checkmark \& \checkmark \& \& \& \checkmark \& \& \& \& \& \checkmark <br>
\hline \checkmark \& \checkmark \& \checkmark \& \checkmark \& \checkmark \& \& \& \& \checkmark \& \& \& \checkmark \& <br>
\hline \checkmark \& \checkmark \& \checkmark \& \checkmark \& \checkmark \& \& \& \checkmark \& \& \& \checkmark \& \& <br>
\hline \checkmark \& \checkmark \& \checkmark \& \checkmark \& \checkmark \& \& \& \& \checkmark \& \checkmark \& \checkmark \& \& \checkmark <br>
\hline \checkmark \& \checkmark \& \checkmark \& \checkmark \& \& \checkmark \& \& \checkmark \& \& \checkmark \& \& \checkmark \& <br>
\hline \checkmark \& \checkmark \& \& \checkmark \& \checkmark \& \& \& \& \& \& \checkmark \& \& <br>

\hline\end{array}\right]\)| p1, p2, p5 |
| :---: |
| new |
| neroducts |

Challenge: Find a ppCA with the minimum number of feature sets

Test Suite Minimization

Software Product Lines

## Background Algorithm Results

## Pairwise Prioritized

 Testing in SPL
## PPGS Algorithm

## Algorithm 1: Pseudocode of PPGS.

```
proc Input:feature model FM, prioritized products prods
    \(\mathrm{TS} \leftarrow \emptyset \quad / /\) Initialize the test suite
    \(\mathrm{RP} \leftarrow\) weighted_pairs_to_cover(prods)
    while not empty (RP) do
        \(\mathrm{t}=0\)
        \(\mathrm{P}(\mathrm{t}) \leftarrow\) Create_Population() \(/ / \mathrm{P}=\) population
        while evals < totalEvals do
            \(\mathrm{Q} \leftarrow \emptyset \quad / / \mathrm{Q}=\) auxiliary population
            for \(\mathrm{i} \leftarrow 1\) to (PPGS.popSize / 2) do
                parents \(\leftarrow\) Selection \((P(t))\)
                offspring \(\leftarrow\) Recombination(PPGS.Pc,parents)
                    offspring \(\leftarrow\) Mutation(PPGS.Pm,offspring)
                    Fix(offspring)
                    ParallelEvaluator.addSolution(offspring)
                    end for
                    solutions \(\leftarrow\) ParallelEvaluator.evaluate();
                    Insert(solutions, Q)
                    \(\mathrm{P}(\mathrm{t}+1):=\) Replace \((\mathrm{Q}, \mathrm{P}(\mathrm{t}))\)
                    \(\mathrm{t}=\mathrm{t}+1\)
            end while //internal loop
            \(\mathrm{TS} \leftarrow \mathrm{TS} \cup\) best_solution \((\mathrm{P}(\mathrm{t}))\)
            RemovePairs(RP, best_solution( \(\mathrm{P}(\mathrm{t}))\) )
        end while //external loop
        return TS
        end_proc
```


## Parameter setting

## Parameter <br> Crossover type <br> one-point <br> Crossover probability <br> 0.8 <br> Selection strategy <br> binary tournament <br> Population size 10 <br> Mutation probability <br> Termination condition <br> Setting

## Background Algorithm Results

## Evaluation

- Compared against Prioritized-ICPL (pICPL)
- Proposed by Johansen et al. (2012)
- Uses data parallelization
- Three different weight priority assignment methods
- Different percentages of selected products
- Ranging from 5\% upto 50\%


## Weight priority assignment methods

## 1. Measured values

- 16 real SPL examples
- Code and feature model available
- Non-functional properties measured (e.g. footprint)

2. Ranked-based values

- Based on how dissimilar two products are
- More dissimilar higher chances of covering more pairs

3. Random values

| SPL Name | Prop | NF | NP | NC | PP\% |
| :--- | :---: | ---: | ---: | ---: | ---: |
| Prevayler | F | 6 | 32 | 24 | 75.0 |
| LinkedList | F | 26 | 1440 | 204 | 14.1 |
| ZipMe | F | 8 | 64 | 64 | 100.0 |
| PKJab | F | 12 | 72 | 72 | 100.0 |
| SensorNetwork | F | 27 | 16704 | 3240 | 19.4 |
| BerkeleyDBF | F | 9 | 256 | 256 | 100.0 |
| Violet | F | 101 | $\approx 1 \mathrm{E} 20$ | 101 | $\approx 0.0$ |
| Linux subset | F | 25 | $\approx 3 \mathrm{E} 24$ | 100 | $\approx 0.0$ |
| LLVM | M | 12 | 1024 | 53 | 5.1 |
| Curl | M | 14 | 1024 | 68 | 6.6 |
| x264 | M | 17 | 2048 | 77 | 3.7 |
| Wget | M | 17 | 8192 | 94 | 1.15 |
| BerkeleyDBM | M | 19 | 3840 | 1280 | 33.3 |
| SQLite | M | 40 | $\approx 5 \mathrm{E} 7$ | 418 | $\approx 0.0$ |
| BerkeleyDBP | P | 27 | 1440 | 180 | 12.50 |
| Apache | P | 10 | 256 | 192 | 75.0 |

Footprint, Main memory consumption, Performance, Number of Features, Number of Products, Number of Configurations, Percentage of Prioritized products.

- [Min..Max] range


## Experimental corpus

|  | G1 | G2 | G3 | Summary |
| :--- | :--- | :--- | :--- | :--- |
| Number Feature Models | 160 | 59 | 16 | 235 |
| Number Products | $16-1 \mathrm{~K}$ | $1 \mathrm{~K}-80 \mathrm{~K}$ | $32-\approx 3 \mathrm{E} 24$ | $16-\approx 3 \mathrm{E} 24$ |
| Number Features | $10-56$ | $14-67$ | $6-101$ | $6-101$ |
| Weight Priority Assignment <br> RK Ranked-Based, RD Random, <br> M Measured | RK,RD | RK,RD | M |  |
| Prioritized Products Percentage | $20,30,50$ | $5,10,20$ | $\approx 0.0-100$ |  |
| Problem Instances | 960 | 354 | 16 | 1330 |

Problem instances G1 = 160 fm X 2 priority assig. X 3 percentages $=\mathbf{9 6 0}$
Problem instances G2 = $59 \mathrm{fm} \times 2$ priority assig. $\mathbf{X} 3$ percentages $=354$
Problem instances G3 = $16 \mathrm{fm} \times 1$ priority assig. $=16$
Total independent runs = $1330 \times 2$ algorithms $\times 30$ indep. runs = 79,800

Software Product Lines

Pairwise Prioritized Testing in SPL

## Background Algorithm Results

## Wilcoxon Test (1)

- Confidence level 95\%
- We show the mean and standard deviation of number of products required to cover $50 \%$ upto $100 \%$ of the total weighted coverage
- We highlight where the difference is statistically significant


## Group G1 - less than $\mathbf{1 0 0 0}$ products

| Cov. | PPGS | pICPL | Cov. | PPGS | pICPL |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $50 \%$ | $1.20_{0.40}$ | $1.20_{0.40}$ | $96 \%$ | $4.00_{1.23}$ | $4.37_{1.42}$ |
| $75 \%$ | $1.92_{0.51}$ | $1.98_{0.58}$ | $97 \%$ | $4.38_{1.32}$ | $4.71_{1.54}$ |
| $80 \%$ | 2.150 .59 | $2.25_{0.68}$ | $98 \%$ | $4.83_{1.46}$ | $5.18_{1.74}$ |
| $85 \%$ | $2.47_{0.72}$ | $2.58_{0.81}$ | $99 \%$ | $5.58_{1.71}$ | $5.87_{1.99}$ |
| $90 \%$ | $2.88_{0.86}$ | $3.13_{1.03}$ | $100 \%$ | $7.56_{2.85}$ | $7.56_{3.03}$ |
| $95 \%$ | $3.72_{1.14}$ | $4.06_{1.33}$ | TIME | $23897_{28669}$ | $10116_{18842}$ |

PPGS smaller size
pICPL faster

## Wilcoxon Test (2)

## Group G2 - from 1,000 to $\mathbf{8 0 , 0 0 0}$ products

| Cov. | PPGS | pICPL | Cov. | PPGS | pICPL |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $50 \%$ | $1.16_{0.36}$ | $1.36_{0.83}$ | $96 \%$ | $4.98_{0.97}$ | $5.83_{3.14}$ |
| $75 \%$ | $2.09_{0.42}$ | $2.47_{1.65}$ | $97 \%$ | $5.55_{1.10}$ | $6.43_{3.27}$ |
| $80 \%$ | $2.39_{0.52}$ | $2.86_{1.79}$ | $98 \%$ | $6.34_{1.34}$ | $7.23_{3.48}$ |
| $85 \%$ | $2.73_{0.59}$ | $3.27_{2.08}$ | $99 \%$ | $7.66_{1.88}$ | $8.59_{4.11}$ |
| $90 \%$ | $3.36_{0.76}$ | $3.98_{2.38}$ | $100 \%$ | $14.57_{10.65}$ | $13.79_{9.98}$ |
| $95 \%$ | $4.59_{0.90}$ | $5.42_{3.12}$ | TIME | $273728_{7.2 E+5}$ | $638164_{2.1 E+6}$ |

- PPGS yields test suites of smaller sizes
- PPGS performs faster than pICPL


## Wilcoxon Test (3) <br> Group G3 - Measured Values, 32 to $\approx 3$ E24 products

| Model | Alg. | 50\% | 75\% | 80\% | 85\% | 90\% | 95\% | 96\% | 97\% | 98\% | 99\% | 100\% | TIME |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Apache | PPGS | 2 | 3 | 3 | 4 | 4 | 6 | 6 | 6 | 7 | 7 | 7 | 10394 |
|  | pICPL | 2 | 3 | 3 | 4 | 5 | 6 | 7 | 7 | 7 | 8 | 8 | 7582 |
| Berk.DBF | PPGS | 2 | 4 | 4 | 5 | 5.97 | 6.97 | 6.97 | 6.97 | 7.97 | 8 | 8.17 | 11213 |
|  | pICPL | 2 | 4 | 5 | 6 | 7 | 8 | 8 | 8 | 8 | 9 | 9 | 8152 |
| Berk.DBM | PPGS | 2 | 3 | 3 | 4 | 4.73 | 6.87 | 7.80 | 8.77 | 9.97 | 11.90 | 23.33 | 117607 |
|  | pICPL | 2 | 3 | 3 | 4 | 6 | 7 | 8 | 8 | 10 | 11 | 21 | 94512 |
| Berk.DBP | PPGS | 1 | 2 | 2 | 3 | 3 | 4 | 4.83 | 5 | 5.93 | 7 | 10.60 | 47361 |
|  | pICPL | 1 | 2 | 3 | 3 | 4 | 6 | 6 | 6 | 6 | 7 | 12 | 57291 |
| Curl | PPGS | 2 | 3 | 3 | 3.97 | 4.03 | 5.83 | 6 | 6.50 | 7.37 | 8.07 | 9.63 | 17454 |
|  | pICPL | 2 | 3 | 3 | 4 | 4 | 6 | 6 | 6 | 7 | 7 | 8 | 6382 |
| LinkedList | PPGS | 1 | 2 | 2 | 2 | 3 | 4.23 | 5 | 5 | 6.13 | 7.79 | 13.37 | 60684 |
|  | pICPL | 1 | 2 | 2 | 3 | 3 | 4 | 4 | 5 | 7 | 11 | 14 | 71151 |
| Linux | PPGS | 2 | 4 | 4 | 5 | 6 | 7 | 7.67 | 8 | 8.37 | 9.40 | 11.10 | 49385 |
|  | pICPL | 2 | 4 | 5 | 5 | 6 | 8 | 8 | 8 | 8 | 9 | 10 | 30522 |
| LLVM | PPGS | 2 | 3 | 3.03 | 4 | 5 | 6 | 6 | 6.07 | 7 | 8 | 8.17 | 12805 |
|  | pICPL | 2 | 3 | 4 | 4 | 5 | 6 | 7 | 7 | 7 | 8 | 8 | 9032 |
| PKJab | PPGS | 1 | 2 | 2 | 3 | 3.07 | 4 | 5 | 5 | 5 | 6 | 7 | 11439 |
|  | pICPL | 1 | 2 | 3 | 3 | 3 | 5 | 5 | 6 | 7 | 8 | 8 | 4661 |
| Prevayler | PPGS | 2 | 3 | 3 | 3 | 4 | 5 | 5 | 5.60 | 6 | 6 | 6 | 8091 |
|  | pICPL | 2 | 3 | 3 | 3 | 4 | 5 | 5 | 5 | 6 | 6 | 6 | 2412 |
| S.Network | PPGS | 1 | 3 | 3 | 3 | 4 | 5.03 | 5.47 | 6 | 6.97 | 7.87 | 13.97 | 71971 |
|  | pICPL | 1 | 3 | 4 | 5 | 6 | 8 | 9 | 9 | 10 | 11 | 17 | 74181 |
| SQL.Mem | PPGS | 1 | 2.17 | 2.90 | 3.23 | 4.07 | 6.14 | 6.97 | 7.93 | 9.23 | 11.70 | 31.53 | 903118 |
|  | pICPL | 1 | 3 | 4 | 4 | 5 | 8 | 8 | 9 | 11 | 14 | 28 | 407991 |
| Violet | PPGS | 1 | 1 | 1 | 2 | 2 | 2.93 | 3 | 3.07 | 3.30 | 4.53 | 12.83 | 31376054 |
|  | pICPL | 1 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 | 6 | 15 | 2471691 |
| Wget | PPGS | 2 | 2.13 | 3 | 3.07 | 4 | 5.43 | 6 | 6.40 | 7 | 8.03 | 11.37 | 31525 |
|  | pICPL | 2 | 3 | 3 | 4 | 4 | 6 | 6 | 7 | 7 | 9 | 11 | 19612 |
| x264 | PPGS | 1.23 | 2.23 | 3 | 3.07 | 4 | 5.30 | 6 | 6.50 | 7.23 | 8.47 | 12.10 | 37368 |
|  | pICPL | 1 | 2 | 3 | 3 | 4 | 5 | 6 | 7 | 7 | 9 | 13 | 13441 |
| ZipMe | PPGS | 2 | 3 | 3 | 4 | 5 | 6 | 6 | 7 | 7 | 7 | 7.03 | 13035 |
|  | pICPL | 2 | 3 | 3 | 4 | 5 | 6 | 6 | 6 | 7 | 7 | 7 | 6142 |

## PPGS

smaller size
faster

Test Suite Minimization

Software Product
Lines

Pairwise Prioritized Testing in SPL

## Background Algorithm Results

## Â12 measure

- $\hat{A}_{12}$ is an effect size measure
- i.e. value 0.3 means that an algorithm A would obtain lower values than algorithm B for a measure M in $70 \%$ of the times
- Lower values, PPGS obtains smaller test suites


PPGS obtains smaller size test suites most of the times

## Recent Research on

## Search Based software Testing: Part 2

Thanks for your attention !!!


Test Suite Minimization

Problem Formulation Landscape Theory Decomposition SAT Transf. Results

Software Product Pairwise Prioritized
Lines Testing in SPL

# Test Suite Minimimation in Regression Testing 

## (Landscape Theory)

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Problem Formulation Landscape Theory Decomposition SAT Transf. Results
2015

## Binary Search Space

- The set of solutions is the set of binary strings with length $\boldsymbol{n}$

| 0 | 1 | 0 | 0 | 1 | 0 | 1 | 1 | 1 | 0 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

- Neighborhood used: one-change neighborhood
$>$ Two solutions $x$ and $y$ are neighbors iff Hamming $(x, y)=1$

$$
\begin{array}{llllllllll}
0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 1 & 0 \\
\hline
\end{array}
$$

| 1 | 1 | 0 |  | 0 | 1 | 0 | 0 | 1 | 1 |  | 1 | 0 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 0 | 0 |  | 0 | 1 | 0 | 0 | 1 | 1 |  | 1 | 0 |
| 0 | 1 | 1 |  | 0 | 1 | 0 | 0 | 1 | 1 |  | 1 | 0 |
| 0 | 1 | 0 |  | 1 | 1 | 0 | 0 | 1 | 1 |  | 1 | 0 |
| 0 | 1 |  |  | 0 | 0 |  | 0 | 1 | 1 |  | 1 | 0 |


| $\mathbf{0}$ | $\mathbf{1}$ | $\mathbf{0}$ | $\mathbf{0}$ | $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{1}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathbf{1}$ | $\mathbf{0}$ |  |  |  |  |  |  |
| $\mathbf{0}$ | $\mathbf{1}$ | $\mathbf{0}$ | $\mathbf{0}$ | $\mathbf{1}$ | $\mathbf{0}$ | $\mathbf{0}$ | $\mathbf{1}$ |
| $\mathbf{0}$ | $\mathbf{1}$ | $\mathbf{0}$ | $\mathbf{0}$ | $\mathbf{0}$ | $\mathbf{1}$ | $\mathbf{0}$ | $\mathbf{1}$ |
| $\mathbf{0}$ | $\mathbf{0}$ | $\mathbf{1}$ | $\mathbf{0}$ |  |  |  |  |
| $\mathbf{0}$ | $\mathbf{1}$ | $\mathbf{0}$ | $\mathbf{0}$ | $\mathbf{0}$ | $\mathbf{1}$ | $\mathbf{0}$ | $\mathbf{1}$ |
|  | $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{0}$ | $\mathbf{1}$ | $\mathbf{1}$ |

## Elementary Landscapes: Characterizations

- An elementary landscape is a landscape for which

$$
\begin{aligned}
& \qquad \underset{y \in N(x)}{\operatorname{avg}\{f(y)\}}=\propto f(x)+(\beta) \forall x \in X \\
& \text { where } \underset{y \in N(x)}{\operatorname{avg}\{f(y)\}} \stackrel{\text { def }}{=} \frac{1}{d} \sum_{y \in N(x)} f(y) \\
& \text { - Grover's wave equation } \\
& \qquad \begin{array}{l}
\operatorname{avg}\{f(y)\}= \\
y \in N(x) \\
\quad \alpha=1-\frac{\lambda}{d}
\end{array} \quad \begin{array}{l}
\text { Depend on the } \\
\text { problem/instance }
\end{array} \\
& \begin{array}{l}
\text { Linear relationship }
\end{array} \\
& \hline X=\frac{1}{d} \sum_{y \in X} f(x)+\frac{\lambda}{d}\langle(\bar{f}-f(x))
\end{aligned}
$$

where

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Pairwise Prioritized Testing in SPL

## Spheres around a Solution

- If $f$ is elementary, the average of $f$ in any sphere and ball of any size around $x$ is a linear expression of $f(x)!!!$



## Landscape Decomposition

- What if the landscape is not elementary?
- Any landscape can be written as the sum of elementary landscapes

- There exists a set of eigenfunctions of $\Delta$ that form a basis of the function space (Fourier basis)



## Elementary Landscape Decomposition of $f$

- The elementary landscape decomposition of

$$
f(x)=\operatorname{cov}(x)-c \cdot \operatorname{cost}(x)
$$

Computable in 0 (nk)
is
Tests that cover $\boldsymbol{e}_{\boldsymbol{i}}$

$$
\begin{aligned}
& f^{(0)}(x)=\sum_{i=1}^{k}\left(1-\frac{1}{2\left(\left|V_{i}\right|\right.}\right)-c \cdot \frac{n}{2} \longleftarrow \text { constant expression } \\
& f^{(1)}(x)=-\sum_{i=1}^{k} \frac{1}{2^{\left|V_{i}\right|}}(-1)^{n_{1}^{(i)}} \mathcal{K}_{\left|V_{i}\right|-1, n_{1}^{(i)}}^{\left|V_{i}\right|}-c \cdot\left(\operatorname{ones}(x)-\frac{n}{2}\right) \\
& f^{(p)}(x)=-\sum_{i=1}^{k} \frac{1}{2^{\left|V_{i}\right|}}(-1)^{n_{1}^{(i)}} \mathcal{K}_{\left|V_{i}\right|-p, n_{1}^{(i)}}^{\left|V_{i}\right|} \quad \text { Where } 1<p \leq n \\
& \text { Krawtchouk matrix }
\end{aligned}
$$

Tests in the solution that cover $\mathbf{e}_{\boldsymbol{i}}$

Test Suite Minimization

## Elementary Landscape Decomposition of $f^{2}$

- The elementary landscape decomposition of $f^{2}$ is

Computable in $O\left(n K^{2}\right)$

$$
\begin{aligned}
\left(f^{2}\right)^{(p)}(x) & =-\sum_{i=1}^{k}\left(\frac{\left(c\left|V_{i}\right|+2 \beta\right)(-1)^{n_{1}^{(i)}}}{2^{\left|V_{i}\right|}} \mathcal{K}_{\left|V_{i}\right|-p, n_{1}^{(i)}}^{\left|V_{i}\right|}\right) \quad p>2 \\
& +\sum_{i, i^{\prime}=1}^{k}\left(\frac{(-1)^{n_{1}^{\left(i v i^{\prime}\right)}}}{2^{\left|V_{i} \cup V_{i^{\prime}}\right|}} \mathcal{K}_{\left|V_{i} \cup V_{i^{\prime}}\right|-p, \text { nin }^{\left(i V^{\prime}\right)}}^{\left|V_{i} \cup V_{i^{\prime}}\right|}\right) \quad \begin{array}{c}
\text { Number of tests in } \\
\text { the solution that } \\
\text { cover } \boldsymbol{e}_{i} \text { or } \mathbf{e}_{i},
\end{array} \\
& -c \sum_{i=1}^{k} \frac{(-1)^{n_{1}^{(i)}}}{2^{\left|V_{i}\right|}} \mathcal{K}_{\left|\left|V_{i}\right|-p+1, n_{1}^{(i)}\right.}^{\left|V_{i}\right|}\left(n-2 \operatorname{mes}(x)-\left|V_{i}\right|+2 n_{1}^{(i)}\right)
\end{aligned}
$$

## Guarded Local Search

- With the Elementary Landscape Decomposition (ELD) we can compute:

$$
\mu_{c}=\underset{y \mid \mathcal{H}(y, x)=r}{\operatorname{avg}\left\{f^{c}(y)\right\}}=\binom{n}{r}^{-1} \sum_{p=0}^{n} \mathcal{K}_{r, p}^{(n)}\left(f^{c}\right)^{(p)}(x)
$$

- With the ELD of $\boldsymbol{f}$ and $\boldsymbol{f}^{2}$ we can compute for any sphere and ball around a solution:

$$
\mu_{1}: \text { the average } \quad \sigma=\sqrt{\mu_{2}-\mu_{1}^{2}} \quad: \text { the standard deviation }
$$

- Distribution of values around the average
$\longrightarrow$ Chebyshev inequality


Apply local s@molt apply local search

## Guarded Local Search: Experimental Setting

- Steady state genetic algorithm: bit-flip ( $p=0.01$ ), one-point crossover, elitist replacement
- GA (no local search)
- GLSr (guarded local search up to radius $r$ )
- LSr (always local search in a ball of radius $r$ )
- Instances from the Software-artifact Infrastructure Repository (SIR)
- printtokens
- printtokens2
- schedule
- schedule2
- totinfo
- replace

Oracle cost c=1..5
$n=100$ test cases
$k=100-200$ items to cover
100 independent runs

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Problem Formulation Landscape Theory Decomposition SAT Transf. Results

## Guarded Local Search: Results



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Pairwise Prioritized Testing in SPL

## Comparison with an LS and GA

## Local Search

Best improvement

## Genetic Algorithm



Total coverage (not Pareto front)

| Instance | Ratio | Algorithm 2 |  | Local Search |  | Genetic Algorithm |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Original (s) | Reduced (s) | Avg. Cov. | Avg. Tests | Avg. Cov. | Avg. Tests |
| printtokens | 4.61 | 3400.74 | 2.17 | 100.00\% | 6.00 | 99.06\% | 5.16 |
| printtokens2 | 4.61 | 3370.44 | 1.43 | 100.00\% | 4.60 | 99.23\% | 3.56 |
| replace | 4.62 | 1469272.00 | 345.62 | 100.00\% | 10.16 | 99.15\% | 15.46 |
| schedule | 2.19 | 492.38 | 0.24 | 100.00\% | 3.00 | 99.84\% | 2.90 |
| schedule2 | 4.61 | 195.55 | 0.27 | 100.00\% | 4.00 | 99.58\% | 3.70 |
| tcas | 4.61 | 73.44 | 0.33 | 100.00\% | 4.00 | 95.80\% | 3.23 |
| totinfo | 4.53 | 181823.50 | 0.96 | 100.00\% | 5.00 | 98.89\% | 5.13 |

